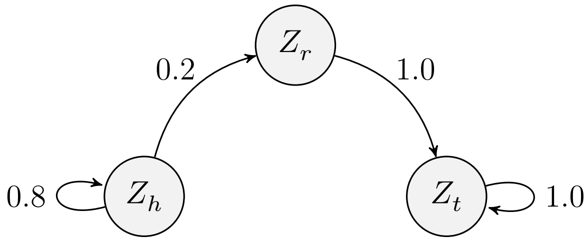
Midterm exam, Genome Informatics 20221025 *Write your name on each sheet.*



	Z_h	Z_r	Z_t
H	0.9	0.5	0.1
T	0.1	0.5	0.9

Emission probabilities of {H,T} for the 3 states.

The model is a "left-to-right" model, always starting in state Z_h.
The observed output sequence is $X =$ HHHTTT.

Notation

Here I suggest some notation to use when showing your work.
Let $\mathrm{P}[X_{1-i}]$ be a shorthand for the first i letters of the output, e.g. $\mathrm{P}[X_{1-4}] =$ HHHT.
Use Q to denote a hidden state sequence, Q_i is the hidden state at time step i. For brevity let Q_{ik} denote $Q_i = Z_k$, e.g. Q_{4r} means the model was in state Z_r in time step 4.
To further simplify notation you may use $\mathrm{P}[h \to r]$ as a shorthand for the transition probability $\mathrm{P}[Q_{i+1} = Z_r | Q_i = Z_h]$.

You will work with numerical values.
We will consider any answer correct in the first 3 digits to be correct.
But please write as many digits as possible, as this can help us trace your work.
(爲了幫助我們確認你的計算，請盡量不要四捨五入)

Problem 1.

What is the probability of the output X given the model, $\mathrm{P}[X|\lambda, \texttt{len}(X) = 6]$?

Problem 2.
What is the maximum likelihood state sequence (Viterbi decoding)?
What is the likelihood of that sequence?

In other words, compute $\mathrm{P}[Q^*|X]$,
where Q^* denotes the maximum likelihood path: $\arg\max_{Q \in \{\mathbf{Q_{1-6}}\}} \mathrm{P}[Q|X_{1-6}]$.
and $\mathbf{Q_{1-6}}$ denotes the set of all state sequences of length 6.

For intermediate calculations, use δ_{ik} to denote: $\max_{Q \in \{\mathbf{Q_{1-ik}}\}} \mathrm{P}[Q|X_{1-i}]$,
where $\mathbf{Q_{1-ik}}$ denotes the set of all state sequences of length i, ending in state Z_k.

Problem 3.
What is the posterior decoding?
In other words, what is the state sequence: $Q^M = Q_1^M Q_2^M \cdots Q_6^M$
where $Q_i^M \stackrel{\text{def}}{=} \max_{k \in \{Z_h, Z_r, Z_t\}} \mathrm{P}[Q_i = Z_k | X]$.

For each position i and state $k \in \{Z_h, Z_r, Z_t\}$, give the probability $\mathrm{P}[Q_i = Z_k | X]$.
Hint: Note that the fact that some transitions have probability one can be used to simplify the backward algorithm computation.